Nominal property based predictive models for asphalt mixture complex modulus (dynamic modulus and phase angle)

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HIGHLIGHTS

• Provides models that use only nominal inputs to make reliable property estimates during design phase.
• Presents generalized regression framework for developing asphalt property prediction models.
• Model is verified through statistical comparisons and comparisons with other predictive models.
• Application of proposed model for pavement performance prediction is demonstrated.

ABSTRACT

Dynamic modulus (|E'|) and phase angle (δ) are necessary for determining the response of asphalt mixtures to in-service traffic and thermal loadings. While a number of |E'| and δ predictive models have been developed, many of them require lab measured properties (e.g. binder complex modulus). The majority of previous work has focused only on prediction of |E'|, limited models exist for prediction of δ. This research utilized generalized regression modelling of lab measurements (from 81 asphalt mixtures) to develop and verify prediction models for |E'| and δ using only nominal asphalt mix properties that are readily available during the initial mixture design and specification process.

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2. Introduction and background

Complex modulus (E') is one of the most commonly used property of asphalt mixtures for conducting pavement analysis and modelling. Two components of complex modulus are, dynamic modulus (|E'|), which describes materials stiffness at given temperature and frequency, and phase angle (δ), which describes the extent of viscous and elastic behavior of the material at a given temperature and loading frequency. Laboratory measurements of |E'| and δ are commonly done at different temperature and frequency combinations using AASHTO T342 procedure. An |E'| master curve is the primary asphalt mixture input in the current AASHTO PavementME design procedure.

Although |E'| and δ can be effectively used to predict the long term performance of asphalt mixtures using mechanistic analysis, there are limitations related to equipment requirements, specimen fabrication complexity, data analysis and other expenses in terms of man-power and time requirements. These limitations have severely restricted wide-spread usage of mechanistic empirical and mechanistic pavement analysis and design. In order to alleviate expensive and time-consuming laboratory testing requirements, a number of predictive equations for |E'| have evolved during the last three decades. Two of the most popular predictive equations for dynamic modulus are the Witczak model [1] and the Hirsh model [2]. Most of these predictive equations are based on regression analysis of large datasets and use the volumetric properties of mixtures along with the binder dynamic shear modulus (G') as their primary input. While there are several models to predict |E'|, there have been far fewer attempts to predict δ.

A distinguishing factor for research and the prediction model presented herein as compared to previous research is that here only nominal properties of asphalt mixtures, such as nominal maximum aggregate size, air void content, asphalt content, the percentage of recycled asphalt pavement (RAP) and recycled...
asphalt shingles (RAS) and asphalt binder performance grade (PG) are used in model development. These parameters are often readily available during the initial phases of asphalt specification and mix design process. The use of PG grades in lieu of other rheological properties of binder like viscosity and complex shear modulus ($G^\prime$) as a continuous factor in the model is logical, since binder PG grade (if not modified) has its own definite rheological characteristics that will impact mixture stiffness. For example, a PG 64-28 in the same temperature and loading condition for a given mix is expected to result in a stiffer (higher $|E^\prime|$) and lower $\delta$ mixture compared to a PG 58-34. This simply means that actual rheological performance of a binder is expressed by the binder's PG grade. Therefore, the information based on PG can be utilized in a predictive model to capture the viscoelastic behavior of the mix. The use of NMAS instead of gradation of the aggregate relies on the fact that any dense graded aggregate with a given NMAS has to be in a specified gradation band to be adopted for construction purposes. Thus, the NMAS itself could be an indicator of the general gradation and can be used as a predictor in the model. Using these simple properties as effective factors in the model, the outcome not only eliminates the need for even simple lab tests, but also provides the pavement design engineers with a tool for specifying asphalt mixture that would yield the best performance and the lowest cost-benefit ratios. The development of phase angle prediction model used the same nominal mix properties as described, with the exception of $|E^\prime|$. This additional variable in prediction of $\delta$ was deemed necessary to be included during the initial model development trials. The existence of this variable is inevitable since $\delta$ is related to $|E^\prime|$ as discussed by Rowe et al. [3] and Oshone et al. [4]. In the initial development of the model, this research used lab measured $|E^\prime|$ values for the prediction of $\delta$, however, the proposed model can effectively use predicted $|E^\prime|$ values.

As one of the most comprehensive equations for prediction of $|E^\prime|$, the Witzczak 2006 model [1] shown below in Eq. (1) is applicable over a wide range of temperatures and frequencies. This model is a revised version of the Witzczak 1999 model in which the viscosity-temperature susceptibility (VTS) method which assumes a linear relationship between temperature and log of viscosity is implemented to characterize the behavior of mixture. This assumption is generally valid for unmodified binders. However, for modified binders it may not be applicable. Thus, this approach could suffer from lack of accuracy when used for characterization of viscoelastic behavior of modified binders [5]. Several studies have been conducted to calibrate these predictive models based on local mixtures and binder types [6]. The Hirsch model alleviates some of these short-comings by using binder $G^\prime$ which is applicable for both modified and conventional binders.

$$P_4 = \text{Cumulative\% retained on \# 4 (4.75 mm) sieve,}$$

$$P_{34} = \text{Cumulative\% retained on 3/8 inch (9.5 mm) sieve,}$$

$$P_{200} = \text{Cumulative\% retained on 3/4 inch (19 mm) sieve,}$$

$$|G^\prime| = \text{Dynamic shear modulus of asphalt binder, (psi),}$$

$$\delta_0 = \text{Phase angle of binder associated with }|G^\prime|, \text{ (degree).}$$

The Hirsch model [2] is based on the Paul's law of mixtures which combines series and parallel elements of the material phases. According to this law, asphalt concrete tends to behave like a series composite at high temperatures and as a parallel composite at lower temperatures. Eq. (2) denotes the Hirsch model for predicting $|E^\prime|$.

$$|E^\prime| = PC \left[ \frac{4200000 \left( 1 - \frac{V_{\text{MA}}}{100} \right)}{V_{\text{MA}}} + 3G^\prime_{\text{binder}} \left( \frac{V_{\text{VFA}}}{V_{\text{MA}}} \right) \right]$$

$$+ \left( 1 - PC \right) \left[ \frac{4200000 \left( 1 - \frac{V_{\text{MA}}}{100} \right)}{V_{\text{MA}}} \right]^{-1} \quad (2)$$

And,

$$P_c = \frac{20 + \frac{V_{\text{FA}}3G^\prime_{\text{binder}}}{V_{\text{MA}}} 0.58}{650 + \frac{20 + \frac{V_{\text{FA}}3G^\prime_{\text{binder}}}{V_{\text{MA}}} 0.58}{650}} \quad (3)$$

where

$|E^\prime|$ = dynamic modulus, (psi)

$G^\prime_{\text{binder}}$ = binder dynamic modulus, (psi)

$V_{\text{MA}}$ = voids in the mineral aggregate, (%)

$V_{\text{FA}}$ = voids filled with asphalt, (%)

$P_c$ = aggregate contact factor

Recently some new approaches have been developed to predict $|E^\prime|$ using artificial intelligence tools and one of them is the Artificial Neural Networks (ANN) method [7]. While this method has shown promising results with a high accuracy of prediction, there are some shortcomings such as, low convergence speed as well as lack of generalizing performance. In other words, even small changes in the input of the model could cause major effects in the model response. Furthermore, they might encounter an overfitting problem [8]. Dynamic modulus has also effectively been predicted using the rheological models like Burger’s and Huet-Sayegh model [9]. Other models have been constructed based on viscoelastic and time-temperature superposition concepts [10]. Finite element based predictive models have been developed to predict dynamic modulus through modeling the effect of random aggregate arrangement during the compaction [11].

$$\log|E^\prime| = -0.349 + 0.754(|G^\prime|^{-0.008})6.65 - 0.032P_{200} + 0.0027P_{200}^2 + 0.011P_4$$

$$- 0.0001P_4^2 ( + 0.006P_{38} - 0.00014P_{38}^2 - 0.08V_\alpha - 1.06 \frac{V_{\text{VFA}}}{V_{\text{VFA}} + V_\alpha})$$

$$+ 2.558 + 0.032V_\alpha + 0.713 \frac{V_{\text{VFA}}}{V_{\text{VFA}} + V_\alpha} + 0.0124P_{38} - 0.0001P_{38}^2 - 0.0098P_{34}$$

$$1 + e^{(0.7814 - 0.5785|G^\prime| - 0.0834 \log |G^\prime|)} \quad (1)$$

where

$|E^\prime|$ = Asphalt mix dynamic modulus (psi),

$V_\alpha$ = Air voids in the mix (% by volume),

$V_{\text{VFA}}$ = Effective binder content (% by volume),

$P_{200}$ = % Passing # 200 (0.075 mm) sieve,

A well-known predictive equation for $\delta$ is based on non-linear regression analysis [12,13]. There are two major limitations to this model, the first being that it uses 16 variables to build up the model that could be decreased. Secondly, this model uses two different regression equations to construct the $\delta$ master curve resulting in a break point.
at the peak value of the master curve which causes non-continuity at that point [14].

This study presents a practical and simple approach to developing $|\varepsilon|$ and $\delta$ prediction models using generalized regression analysis. Using this approach, $|\varepsilon|$ and $\delta$ models that utilize only nominal properties of HMA mixtures have been developed for New England region of the United States. These properties are readily available during any preliminary mixture design procedure, which means that there would be no requirements for any type of lab tests. The attributes of the mixtures used in this study as well as a brief description of generalized regression platform and model development is discussed next in the paper. The predictive models are evaluated statistically and their actual field applicability was assessed using a case study, these are presented later in the paper.

3. Research approach and materials

This study utilized 81 asphalt mixtures with diverse volumetric and constituent properties. All mixtures were designed according to the Superpave procedure [15] and tested following the AASHTO T342 [16] procedures in unconfined condition using an Asphalt Mixture Performance Tester. The mixtures represent materials from the New England region of the United States. Each test was conducted with three replicate specimens tested at three temperatures and six loading frequencies. Among the whole dataset, there are 27 mixtures that have been manufactured with the usage of modified binder and implementing the warm mix asphalt (WMA) production technology. The mixture attributes and the test parameters were presented in Table 1.

Along with the variable selection for the predictive models, principal component analysis (PCA) was performed on the correlations of variables and lab measured values of $|\varepsilon|$ and $\delta$. Correlation values from this analysis are presented in Table 2. The values shown in the table indicate the dependence of one factor on another in a numerical manner. Negative correlation coefficients indicate that the two variables are inversely dependent on each other. This type of analysis allowed for better understanding the relationship between the selected factors and the responses. Initially, it might appear that the correlations of individual independent variables with the $|\varepsilon|$ and $\delta$ is low. It should be noted that in asphalt mixtures, which are composite materials, it is the interaction of the individual variables that has a significant effect on both $|\varepsilon|$ and $\delta$. In other words, $|\varepsilon|$ and $\delta$ are direct functions of a mix design which is a combination of all the variables (Va, AC, NMAS, RAP, RAS etc.). Therefore, a general expectation of a high correlation between the response and individual mix related variables might not be true. For example, it was observed that the PGHT did not have a significant effect on the model whereas its interaction with the logarithm of temperature is an effective term. Additionally, using these uncorrelated factors helps avoid the multi-collinearity problems, which might cause erratic p-values for the independent variables as well as incorrect relationship between the predicted response and the predictors. On the other hand, the temperature and loading frequency (referred to as the test related variables) can individually affect the viscoelastic behavior of the mix [17] and as Table 2 shows these two variables have high correlations with the response. The significance of the variable interactions is presented in the model development section.

The analysis to build predictive models presented in this study was conducted using JMP PRO software which is a statistics analysis tool. Generalized regression platform allows for fitting of penalized generalized linear models to data sets. The models are penalized in the sense that a penalty is added to the likelihood of the model. The penalizing equivalently constrains the sum of absolute values of the estimates and causes some of them to turn out to be zero, which helps in eliminating the redundant variables. Depending on the form of the penalty, this allows variable selection as well as shrinkage of estimators. Generalizing the models eliminates the need of normal distribution of response. This is useful, since in many instances there are responses which are not normally distributed. Generalized regression approach uses relationships between the dependent and independent variables using coefficients that can vary with respect to one or more grouping variables for non-normally distributed situations [18].

Use of penalized regression also helps in lowering the number of effective terms in a model. In standard linear regression, having higher numbers of effective terms can easily cause an issue that is commonly referred to as overfitting. This means that the model will fit the observed data very well, but it will perform poorly on new observations. If a model is optimized by penalization, there would be certain benefits such as the better prediction of data by avoiding overfitting, as well as easier interpretation of the resultant model. The two main penalization methods are the “Lasso” and “Elastic-net”; both of these methods shrink some predictors to a nil or zero value. The Lasso method will tend to give a more parsimonious model than the elastic-net, while the Elastic-net can better handle collinearity than the lasso. Simulation studies and real data examples show that the Elastic-net method often outperforms the Lasso method in terms of prediction accuracy. In

| Table 1: Scope of mixture attributes and test parameters in the dataset. |
| --- | --- | --- | --- | --- |
| Predictive Model | Type of Factor | Name of Factor | Abbreviation | Number of Levels | Level (Number of mixes) | Min | Max |
| E’ and $\delta$ | Mixture Related | Nominal Maximum Aggregate Size (mm) | NMAS | 3 | 9.5 (37), 12.5 (35), 19 (9) | – | – |
| | | Air Void of Total Mixture (%) | Va% | 23 | – | 3% | 9.63% |
| | | Total Asphalt Content (%) | AC% | 14 | – | 4.70% | 6.80% |
| | | Recycled Asphalt Pavement (%) | RAP | 13 | – | 0% | 40% |
| | | Recycled Asphalt Shingle (%) | RAS | 2 | 12.2 (4), 11.1 (2) | – | – |
| | | Binder High Temperature PG grade (°C) | PGHT | 3 | 64 (49), 58 (16), 52 (16) | – | – |
| | Test Related | Binder Low Temperature PG grade (°C) | PGLT | 3 | -34 (16), -28 (61), -22 (4) | – | – |
| | | Logarithm of Loading Frequency (Hz) | Log (Frequency) | 6 | 0.1, 0.5, 1, 5, 10, 25 (81 at each) | – | – |
| | | Logarithm of Test Temperature (°C) | Log (Temperature) | 3 | 3.9, 20, 35 (29); 4.4, 21.1, 37.8 (52) | – | – |
| $\delta$ | Material Mechanical Property | Logarithm of Dynamic Modulus | Log($\varepsilon$) | – | – | – | – |
addition, the Elastic-net encourages a grouping effect, where strongly correlated predictors tend to be in or out of the model together [18,19]. The Elastic-net procedure was selected in this study as a penalization method due to its superiority in terms of variable selection and prediction accuracy. Optimal penalty values can be determined using different validation methods. In the present work, Bayesian Information Criterion was used due to its computational efficiency and ability to result in a parsimonious model [19].

4. Development of dynamic modulus (|E'|) and phase angle (δ) prediction models

Prior to determination of the predictive models, the Mahalanobis distance analysis [20] was conducted on the whole dataset to assess data quality and to statistically identify outliers. Using this analysis, 237 data-points were excluded from the analysis resulting in a substantially unified dataset and an unbiased predictive model. Since |E'| is considered to be one of the variables for δ prediction model, the same outliers were also omitted in δ model development. The procedure to develop the model requires response distribution data to be provided to the statistical software. Fig. 1 represents the distribution shapes for the logarithm of |E'|, |E'| and δ. Although the shapes of logarithm of |E'| and δ can be considered as gamma distributions, further trial and error attempts proved that using the normal distribution slightly improved the RMSE (Root Mean Squared Error) and goodness of fit (R²).

The next step in developing the model was to examine diverse types of combinations of factors to evaluate their significance as well as practical interpretability. Using this iterative process, the prediction models were developed. The prediction models for |E'| and δ are depicted in Tables 3 and 4 respectively. It can be inferred from the |E'| model that the test related factors and their interaction along with the quadratic effect of the logarithm of temperature have the highest impact on |E'|. Consequently, the mixture related factors are observed to have significant effects on |E'| except for the binder high PG temperature (PGHT). The |E'| and δ distribution plots indicate that more than 25% of the observations on |E'| range from 100 to 1000 MPa and for δ from 30 to 50°. These values normally indicate the results from the higher test

Table 2
Correlation matrix of variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Log(ΔE*(lab measured))</th>
<th>Phase Angle</th>
<th>Log(Temperature)</th>
<th>Log(Frequency)</th>
<th>AC%</th>
<th>NMAS</th>
<th>Va%</th>
<th>RAP%</th>
<th>RAS%</th>
<th>PGHT</th>
<th>PGLT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log(ΔE*(lab measured))</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Phase Angle</td>
<td>–0.67</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Log(Temperature)</td>
<td>–0.79</td>
<td>0.75</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Log(Frequency)</td>
<td>0.40</td>
<td>–0.12</td>
<td>0.07</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>AC%</td>
<td>–0.06</td>
<td>–0.09</td>
<td>–0.06</td>
<td>0.01</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>NMAS</td>
<td>0.04</td>
<td>0.13</td>
<td>0.04</td>
<td>–0.01</td>
<td>–0.85</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Va%</td>
<td>–0.07</td>
<td>–0.01</td>
<td>–0.02</td>
<td>0.02</td>
<td>0.07</td>
<td>–1.14</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>RAP%</td>
<td>0.06</td>
<td>0.06</td>
<td>0.02</td>
<td>–0.02</td>
<td>–0.21</td>
<td>0.23</td>
<td>–0.08</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>RAS%</td>
<td>0.003</td>
<td>0.05</td>
<td>0.01</td>
<td>0.01</td>
<td>–0.38</td>
<td>0.49</td>
<td>–0.08</td>
<td>–0.12</td>
<td>1.00</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>PGHT</td>
<td>0.06</td>
<td>–0.09</td>
<td>0.02</td>
<td>–0.02</td>
<td>0.27</td>
<td>–0.36</td>
<td>0.11</td>
<td>–0.36</td>
<td>–0.32</td>
<td>1.00</td>
<td>–</td>
</tr>
<tr>
<td>PGLT</td>
<td>0.11</td>
<td>–0.06</td>
<td>0.06</td>
<td>–0.02</td>
<td>–0.03</td>
<td>–0.13</td>
<td>0.10</td>
<td>–0.14</td>
<td>–0.21</td>
<td>0.83</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Fig. 1. Distributions of (a) Logarithm of |E'|, (b) |E'| and (c) δ data used in model development.
temperatures. The main load-bearing component at higher test temperature (low loading frequency) is the aggregate. Usually aggregates used in HMA mixtures have similar mechanical properties that result in less variations in the observed values of $|E^*|$ and $\delta$ at higher temperatures. This is hypothesized to be the cause for lower significant factor related to asphalt PGHT. However, both predictive models show that the interaction of PGHT with logarithm of temperature and logarithm of loading frequency are effective and that is another reason to keep the PGHT in the model. The reverse state happens for asphalt binder PG low temperature (PGLT) where there is a large variation in observed $|E^*|$ and $\delta$ values at lower testing temperatures. The analysis also shows that this factor has a high impact on both models. The predictive values of $|E^*|$ and $\delta$ can be calculated through the coefficients and equations shown in Table 3. Since the predictive models are based on the generalized regression, which is different from multiple regression analysis, the interaction of variables along with the independent variables can be effectively used. The $|E^*|$ predictive model has been built upon the logarithm of measured dynamic modulus which has resulted in small coefficient values in the model.

Measurements of $\delta$ in the lab are usually challenging and there is a higher variability associated to this mixture property, which makes the construction of a reliable and accurate predictive model more challenging. This resulted in the usage of a much higher...
number of effective terms in the model to increase the level of accuracy, which is still lower than the accuracy of the $|E'\over\omega|$ model.

5. Evaluation of prediction model capabilities

To verify the $|E'|$ and $\delta$ prediction models, a set of analyses were conducted to compare predicted values with lab measured values. The comparisons are made on unity plots where predicted and measured values are plotted against each other for the whole dataset. The "goodness of fit" statistics parameters such as the correlation coefficient ($R^2$) and RMSE were calculated. The correlation coefficient indicates how well the regression line approximates the measured data points. RMSE is a way to measure the difference between predicted and measured values in a prediction model.

Fig. 2 shows the goodness of fit statistics for predicted $|E'|$ (log and arithmetic) and $\delta$ with measured values respectively. A very high $R^2$ and low RMSE for linear fit between predicted and measured $|E'|$ values on log-log plot and the equation of linear fit to be very close to unity slope line ($X = Y$) reveals that model is fitted very well to the measured data. A small deviation in the linear fitting line for the predicted and measured $|E'|$ (for both log-log and arithmetic scales) comes from the software number rounding when the log scale is changed into the arithmetic one. The actual values of $|E'|$ range from 100 to 12000 MPa, a RMSE of 833.8 expresses an average of 7% difference between the predicted and actual $|E'|$ values, this is well below typical laboratory variability.

A high $R^2$ of 0.83 for $\delta$ predictive model ranks it among the good correlations between the actual observations and the predicted values. The lab measured values of $\delta$ range from 9° to 45° and a RMSE of 2.94° expresses an average of 8% difference between the predicted and lab measured $\delta$ values in the dataset.

In addition to comparing the complete datasets between measured and predicted values of $|E'|$ and $\delta$, further investigations were conducted for 6 mixtures. The mixtures were chosen to be significantly different from each other in terms of constituents and the lab measured $|E'|$ and $\delta$. The properties of these mixtures are shown in Fig. 3.

Master curves of $|E'|$ and $\delta$ were constructed at 21.1°C for Mixtures A-D and at 20°C for Mixtures E-F. All the shift factors were obtained by using a second order polynomial shift factor equation using rheological and viscoelastic analysis software RHEA [21].

The sample standard deviation for each replicate at each test temperature and frequency was also calculated to obtain the high and low range of measured $|E'|$ and $\delta$. For each set of measured data (average, average +1 standard deviation and average -1 standard deviation), independent time-temperature shifting was conducted and this yielded three master curves for $|E'|$ and $\delta$ from lab data. These will be referred to as "Measured", "Measured High Range" and "Measured Low Range" throughout the remainder of this report.

A four parameter logistic regression sigmoidal equation was used to fit shifted data for constructing $|E'|$ master curves. The fitting equation is shown below.

$$\log(|E'|) = c + \frac{d - c}{1 + e^{-a(\log(f) - b)}}$$ (5)

where

- $f$ = Load Frequency
- $a$ = Growth Rate
A Lorentzian peak equation was used to fit the shifted phase angle results to construct the $d$ master curves.

$$d = \frac{a}{c^2 + b^2}$$

where

- $f$ = Load Frequency
- $a$ = Peak Value
- $b$ = Growth Rate
- $c$ = Critical Point

In order to calculate quantifiable differences between master curves from lab measurements and model predictions, sum of squared errors (SSE) were calculated for each of the six mixtures for both $|E'|$ and $\delta$. Eleven frequencies (0.001, 0.01, 0.1, 0.5, 1, 5, 10, 25, 100, 1000 and 10,000 Hz) were selected for SSE calculations. Also, for the purpose of visual comparisons of $|E'|$ and $\delta$ on single plots, Black space diagrams have been prepared. The comparison plots for $|E'|$, $\delta$ and Black space are presented in Figs. 3–5 respectively.

Using the values of SSE/n statistics, the comparison plots of $|E'|$-for the six mixtures show that the prediction equation for majority of mixtures yields values that are close to lab measured values and often within lab measurement variability. A majority of the deviation between measured and predicted $|E'|$ is observed at very low frequencies. At these frequencies, the $|E'|$ response of asphalt mixture is often dominated by aggregate skeleton; the model presented here does not take into account aggregate size distribution and thus a small discrepancy is expected in this region. Overall, the SSE/n values are quite low indicating that the model predictions are quite close to $|E'|$ from lab results.

Considering the SSE/n values in case of $\delta$, the model predictions at a majority of frequency ranges for all mixtures is close to the master curve from lab measurements. As with $|E'|$, there is some variation observed between predicted phase angle master curves and those generated using lab data in the lower frequency ranges. The differences are typically in the range of 2 to 5°, while typical lab variability of this measure is also about 5°. The average SSE/n values for all six mixtures are also relatively low with the highest
being 107.6, which indicates average distributed prediction error of 10.4°. As described before, one major advancement in the current research over previous research is the $\delta$ prediction model. The majority of current $E'$ prediction equations do not provide phase angle prediction and the ones that do provide it require viscoelastic characterization of binder for accurate prediction. In order to fully describe viscoelastic behavior of asphalt materials and to accurately calculate stress and strain response at different service temperatures and at different loading frequencies, it is critical to have $\delta$ master-curve.

In recent years, Black space diagrams have been used for comparison of asphalt mixture performances, for example, work by Mensching et al. [22]. In order to compare the model predictions with lab measurements for both $E'$ and $\delta$, Black space diagrams have been generated for the six mixtures discussed here (see Fig. 5). The plots show very similar Black space response for model predictions and lab measured data. Thus, if Black space based performance prediction parameters are used for performance based specifications, the models proposed herein can be easily utilized for determining these parameters during the mix design stage.

Among the evaluated mixtures, C and D reveal a larger difference between the measured and predicted values of phase angle and this could be due to the usage of modified binder as well as implementing the warm mix asphalt (WMA) technology in manufacturing process of these mixes. Even so, with the use of WMA and modifiers, the predicted $E'$ for these mixtures is close to the actual lab measurements.

6. Fatigue Performance Analysis Using Predicted Properties

To demonstrate the ability of the prediction models for purposes of pavement performance evaluation, a brief case study was conducted. This was also driven by the underlying intent of this research, which is to implement $E'$ and $\delta$ prediction models for determination of the pavement performance as a combined asphalt mixture and pavement design tool. The case study used lab measured and predicted $E'$ and $\delta$ values within simplified viscoelastic continuum damage (S-VECD) framework for fatigue cracking performance evaluation. While the research presented here is very useful for conducting PavementME designs and analysis during mix design and selection phase, this research predicts
more comprehensive mixture characterizations vis-a-vis $|E'\|/\delta$ than what is needed for PavementME. For brevity only one mixture (Mixture A) from the previous section was selected for the fatigue performance analysis.

The lab measured results of uniaxial fatigue testing from the selected mixture along with the $|E'\|/\delta$ (lab measured and predicted) were used as the principal inputs for SVECD analysis. SVECD analysis resulted in damage characteristic curves (DCC) for mixture, DCC indicates the relationship between the asphalt mixture's material integrity (called the Pseudo stiffness ($C$)) and the level of damage over time ($S$) [23]. DCC were calculated using both measured and predicted $|E'\|/\delta$.

While DCC is an indicator of how well the mixture can bear the applied loads and how damage progresses with repeated loading, the actual performance of a mixture also depends substantially on the pavement cross section, climatic conditions and material constitutive properties. In order to determine the pavement performance, an investigation was conducted using the layered viscoelastic pavement analysis for critical distresses (LVECD). This program utilizes the SVECD model to calculate the rate of strains and stresses over the life of pavement to make the performance predictions [24]. During recent years this software has been widely used by many researchers in predicting the fatigue performance of asphalt mixtures as well as determining how mix parameters affect its performance in actual field situations, for example recent work by Rastegar et al. [25]. One of the main results from LVECD analysis is damage factor, this factor simply reveals that how much of a cross section has been damaged due to loading and other factors leading to pavement deterioration over time. Using Miner's law, the number of points (evenly distributed regions of asphalt concrete over the simulation domain) where the damage factor is equal to one, or where asphalt concrete has fully damaged, is calculated over life of pavement.

Two types of cross sections were analyzed using LVECD. Only the thickness of the asphalt layer was changed in these cross sections. Fig. 6 presents the DCC and LVECD analysis results. As it can be seen from the figures, both measured and predicted $|E'\|/\delta$ led to very comparable DCC. Additionally, in the context of pavement performance evaluation, the predicted results are not identical between measured $|E'\|/\delta$ and $\delta$ and predicted ones. However, the results are very comparable with each other. The pavement performances using the predictive $|E'\|/\delta$ and $\delta$ values at 20 years are very comparable to the performance predicted using lab measured values for both thin and thick asphalt pavements. Thus, fatigue performance calculations from the predicted $|E'\|/\delta$ and $\delta$ are comparable to the measured ones for both cross sections, which is a good indicator of the applicability of the predictive models presented in this paper.

7. Comparison of the $|E'\|$ predictive model to the Witczak model and Hirsch model

As the final step in the evaluation of the accuracy of the predictive models presented in this study, the $|E'\|$ predictions are com-
pared to the Witczak model and Hirsch models. The comparison is made for Mixture B. As indicated in previous discussion, both Witczak model and Hirsch model require asphalt mix properties that may not be readily available during the mix design stage. Please note that there are different versions of Witczak model for dynamic modulus prediction, the newest published version of the model, Witczak 2006 [1], is used in this work. In this study, comparisons are made to For example, the results shown in Fig. 7 required asphalt binder complex shear modulus ($G'$) at different temperatures for both Witczak model and Hirsch model predictions. Measured values for the binder were used in this study to make the predictions. Fig. 7 indicates that even with lab measured binder properties both Witczak model and Hirsch model substantially over-predicted $|E'|$ at lower load frequencies. The generalized regression based model from the present study yielded $|E'|$ master-curve to be very close to that generated from the lab measurements.

8. Summary, conclusion and future extensions

This paper describes generalized regression based prediction models for dynamic modulus ($|E'|$) and phase angle ($\delta$) of asphalt mixtures. The models were developed using over 4300 laboratory test results for asphalt mixtures in the New England region of the United States. A unique feature of the model development approach, as well as the models presented here are that they utilize nominal asphalt mixture properties. This is different than currently adopted prediction models, and thus make proposed models very useful for pavement designers. For example, using the models proposed here, a pavement designer can conduct mechanistic pavement analysis and make recommendations for modifying asphalt mixture specifications for a given project and/or optimize the pavement structure and material to lower life cycle costs. As previously mentioned, the models proposed here have been developed using the dataset gathered in the New England region, therefore the applicability of the regression coefficients presented herein may be limited to this region due to similarities in aggregate geological sources, binder grades and recycled asphalt material characteristics. However, the methodology behind the development of these models can be applied to other regions for development of regional prediction models. Although the dataset has been gath-
ered for complex moduli of asphalt mixtures, it does not necessarily mean that the development of such models is associated to only complex moduli and only measurements made using AMPT device. In fact, the framework presented in this study can be applied to develop similar models for other material properties. It is also important to note that the use of prediction models does not necessarily result in full omission of conducting \( E' \) and \( \delta \) lab tests, rather predictive models aid in lowering the amount of testing requirements as well as provide reliable estimate of properties when lab testing is impractical or not possible due to time or economic constraints.

A practical application of the proposed model is for developing asphalt specifications and for conducting pavement structural design. At present, a major hurdle in developing asphalt mix specifications on basis of mechanistic properties and conducting pavement structural designs using properties that reliably represent the actual mixture that will be produced and placed in the field, is unavailability of reliable prediction models that only use nominal properties to predict \( E' \) and \( \delta \). While Hirsch model and Witzczak model have been adopted, these require binder viscoelastic characterization for prediction. Furthermore, as shown in this paper, even with binder viscoelastic characterization these models can fail to make reliable predictions.

Comparisons were made between lab measured data and the model predictions. While the same data was used for developing the model, this comparison provides verification of the model development process. Apart from visual comparisons, statistical comparisons were also conducted. To further ensure veracity of the models, six distinctly different mixtures were chosen and comparisons were made between model predicted and lab measured \( E' \) and \( \delta \). The predictions were mostly within one standard deviation lab variability of measured quantities. Finally, to demonstrate the application of the prediction model and to make further comparisons between model predictions and lab measurements, a case study is presented for two asphalt pavement cross-sections and their predicted fatigue cracking performances. The results from this analysis demonstrate that the model predictions presented herein are capable of use in pavement mechanistic analysis tools and yield comparable results to those from lab measured properties.

On the basis of the research results presented in this paper, the following conclusions can be drawn:

- Generalized regression based methodology can be employed for developing dynamic modulus and phase angle prediction models that require only nominal asphalt mixture parameters as inputs;
- The predictions from generalized regression based models match the lab measurements within typical lab variability;
- Rheological indices for pavement performance can be easily calculated using the prediction models presented here, these indices can be used for performance based specifications; and,
- Using prediction models presented herein, pavement designers can optimize asphalt mix specifications to increase reliability of pavement designs and to lower life cycle costs.

While this paper presented prediction models, their comparisons with lab measurements and a case study to demonstrate applicability of the models, there were several areas identified during this research that will further improve the applicability of this research and extend the findings further. Some of the future extension of the present research are the following:

- The current models are developed for New England region; similar regional models can be developed for other part of United States and other countries. Notice that it is important to try to limit these type of models to a region, that way only nominal asphalt properties would be necessary as model inputs, otherwise the required number of inputs might become overwhelming.
- In this work, the generalized regression based model development was applied to linear viscoelastic asphalt properties; future efforts should undertake similar model development for non-linear properties such as asphalt fatigue and fracture parameters.
- Validation of the predictions models should be conducted using additional mixtures that are not part of the model development data set. Furthermore, field performance validation should also be conducted.
- The other aspect of the future work is the improvement of the accuracy of the proposed models and especially the \( \delta \) by using additional number of mixtures and calibrating the models in accordance to the newly added data.
- Future predictive models can be developed within framework of analytical/physical models so that such model can therefor be incorporated within mechanistic calculation algorithms.

**References**


