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To cite this article: Mirkat Oshone, Eshan Dave, Jo Sias Daniel & Geoffrey M. Rowe (2017): Prediction of phase angles from dynamic modulus data and implications for cracking performance evaluation, Road Materials and Pavement Design, DOI: 10.1080/14680629.2017.1389086

To link to this article: http://dx.doi.org/10.1080/14680629.2017.1389086

Published online: 30 Oct 2017.
Prediction of phase angles from dynamic modulus data and implications for cracking performance evaluation

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(Received 15 August 2016; accepted 25 October 2016)

The need for a viscoelastic characterisation of hot mix asphalt is increasing as advanced testing and modelling is incorporated through mechanistic-empirical pavement design and performance-based specifications. Viscoelastic characterisation includes measurement of the mixture stiffness and relative proportion of elastic and viscous response. The most common method is to measure the complex modulus, where dynamic modulus represents the stiffness and the phase angle represents the relative extent of elastic and viscous response. Determination of phase angle from temperature and frequency sweep tests has been challenging, unreliable and prone to error due to a high degree of variability and sensitivity to signal noise. There are also large amounts of historical dynamic modulus data that are either missing phase angle measurements or have poorly measured phase angle data that inhibit their use in further evaluation. This paper evaluates the robustness of phase angle estimation from stiffness data for asphalt mixtures. The objectives of the study are to: (1) evaluate the procedure of estimating phase angle from the slope of log-log stiffness master curve fitted with a generalised logistic sigmoidal curve and compare it with lab measurements and the Hirsch model; (2) assess the effect of measured and predicted phase angles on a mixture Black Space diagram; (3) evaluate the effect of using predicted phase angles on SVECD fatigue analysis particularly regarding damage characteristics curves and fatigue coefficients and (4) evaluate the impact on layered viscoelastic pavement analysis for critical distresses (LVECD) pavement fatigue performance evaluation due to the use of predicted phase angles. Three sets of independent mixtures were evaluated in this study comprising a wide range of mixture conditions. The results indicate good agreement between measured and predicted phase angle values in terms of shape and peak master curve values. In terms of magnitude, the values from both matched very well for certain sets of mixtures and subsequently manifested in similar performance predictions. However, for other sets of mixtures, a considerable difference was observed between measured and predicted phase angle values as well as SVECD and LVECD results. The differences may be attributed to the use of different types of linear variable displacement transducers (loose core versus spring loaded). Another possible explanation for the difference could be the contribution of plastic strain, which may create a difference in phase angles of 1–2°.

Keywords: asphalt mixtures; slope of dynamic modulus master curve; phase angle master curve; rheological indices; fatigue performance

Introduction and background

Asphalt binders exhibit aspects of both elastic and viscous behaviours, and therefore they are considered as viscoelastic materials. Viscoelastic materials are some of the most common materials that we encounter and are frequently used for many engineering applications. The mechanical
properties of viscoelastic materials are temperature, frequency, loading history and being time-dependent. Most often the creep and flow behaviour is small and can be neglected in engineering computations. However, asphalt binders and mixtures need to be fully characterised to capture the viscoelastic behaviour to understand the performance of pavement structures. Viscoelastic materials exhibit both elastic and viscous characteristics during load application. The elastic component of the response is described by the storage modulus and the viscous component of the response is the loss modulus. It is important to accurately measure both components of the response; however, the complex nature of the mechanical behaviour presents experimental difficulties and uncertainties during material characterisation.

Asphalt mixtures manifest a more complex viscoelastic behaviour due to the combination of the viscoelastic asphalt binders and the aggregate skeleton. Researchers have shown that asphalt mixtures demonstrate linear viscoelastic properties within a small strain level (<100 micro-strain) and a limited number of cycles (Airey & Rahimzadeh, 2004; Gardner & Skok, 1964). Yet, some studies argue that nonlinear viscoelastic behaviour can appear at strain levels as low as 40 micro-strain (Sayegh, 1967). For materials that exhibit linear viscoelastic behaviour, the relationship between stress and strain depends on loading frequency and temperature and can be fully described using a complex modulus (dynamic modulus and phase angle); this test is straightforward and easy to adopt for the characterisation of asphalt mixes in the small strain region. Moreover, when asphalt mixtures are in the linear viscoelastic range, they generally exhibit thermorheologically simple properties. The time–temperature superposition principle can then be employed to horizontally shift results measured at different temperatures along the time or frequency axis to construct a master curve for the full characterisation of material behaviour (Van der Poel, 1955). The amount of time or frequency shift is called the time–temperature shift factor. By combining the master curve with the shift factor, it is possible to predict the linear viscoelastic behaviour of materials over a wide range of frequency and temperature conditions.

The complex modulus test has been one of the methods in use for linear viscoelastic characterisation of asphalt mixtures in undamaged states since the 1950s (Heukelom & Klomp, 1964; Van der Poel, 1955). This is achieved by determining two fundamental viscoelastic properties, namely, dynamic modulus, \( |E^*| \) and phase angle. Based on comparative studies, Elseifi, Al-Qadi, and Yoo (2006) concluded that inclusion of a viscoelastic constitutive model into pavement design methods leads to improved accuracy. Currently, different structural and performance mechanistic models use dynamic modulus and phase angle master curves for linear viscoelastic characterisation of asphalt mixtures at a required range of temperature, strain rates and stress states. Specific applications include the determination of various parameters including binder or mix rheological parameters – such as R-value (Christensen & Anderson, 1992), the Glover–Rowe (G–R) parameter (King, Anderson, Hanson, & Blankenship, 2012; Rowe, King, & Anderson, 2013), inflection point frequency, mixture Black Space plots (Mensching, Rowe, & Daniel, 2017), the C1 and C2 parameters in the Williams Landel Ferry (WLF) equation (Rowe, Baumgardner, & Sharrock, 2009; Kaelble, 1985), and lower and upper asymptote of mix master curves with a sigmoidal form. The application also extends to fatigue characterisation and performance prediction models such as simplified viscoelastic continuum damage (SVECD) and layered viscoelastic pavement analysis for critical distresses (LVECD).

The mechanistic analysis of pavements greatly depends on the material characterisation method and its accuracy. In recent years, significant advances have been made in specimen fabrication and testing equipment resulting in increased precision of the test data and lower variability associated with \( |E^*| \) measurements. Moreover, well-developed and robust \( |E^*| \) prediction equations such as the Witczak model (Andrei, Witczak, & Mirza, 1999), Hirsch model (Christensen, Pellinen, & Bonaquist, 2003) and others are available and have been successfully used by researchers. The long-term pavement performance (LTPP) programme has also employed these
models to determine $|E^*|$. While it is known that the accurate measurement of phase angle is very important for determining the elastic and viscous components, the measurement of phase angle in the laboratory still remains a challenge due to the need to accurately capture time-based data with existing measurement technology. The variability mostly arises from the large amount of inherent noise in the deformation measurement signal. In addition, the calibration aspect is also complex. Generally, a testing device is evaluated using a solid fixture for a zero-phase lag response. However, standards of materials with a known stiffness and intermediate values of phase angle (typical of that found in asphalt mixtures) are not available. The accuracy further depends upon many other factors (e.g. adjustments for equipment compliance that have been made, details of displacement transducer design). Moreover, large amounts of historic data exist with $|E^*|$ measurements but with no accompanying phase angle measurements. For example, the LTPP database is populated with measured and predicted $|E^*|$ data but lacks phase angle ($\phi$) data. This prohibits the use of these data for rheological and performance evaluation. Further, it inhibits their use for verification of rheological parameters and linkage of historic data to field performance data.

Several researchers have developed relationships between phase angle and modulus for asphalt mixtures. Bonnaure, Gest, Gravois, and Uge (1977) developed a relationship that was limited to binder stiffness ($S_b$) values greater than 5 MPa and less than 2 GPa (when $S_b$ is greater than 2 GPa the mixture phase angle ($\phi_m$) is taken to be zero). The relationship used the binder stiffness ($S_b$) volume of binder ($V_b$) in the prediction and is as follows:

$$\phi_m = 16.36V_b^{0.352}\exp\left[\frac{\log_{10}S_b - \log_{10}5 \times 10^6}{\log_{10}S_b - \log_{10}2 \times 10^9} \times 0.974V_b^{-0.172}\right].$$

(1)

During the SHRP project, Tayebali, Tsai, and Monismith (1994) developed several relationships linking phase angle to mix stiffness, an example of which is shown below:

$$\phi_o = 260.096 - 17.172\ln(S_o),$$

(2)

where $\phi_o$ is the mixture phase angle and $S_o$ is the mixture stiffness. This relationship was developed from a study of fatigue properties. The subscript to the parameters denotes that the initial condition is used. The preceding two relationships for bituminous mixtures are empirical in nature with derived constants from regression analysis.

The Hirsh model was originally developed in the late 1960s based on the modified law of mixtures. The law states that the property of a composite material can be treated as a combination of the properties of its components assuming the influence of each component is proportional to its volume fraction. Christensen et al. (2003) developed a calibrated phenomenological model based on the Hirsch model that links phase angle to binder properties and mixture volumetrics, as follows:

$$|E^*|_{mix} = Pc \left[4,200,000 \left(1 - \frac{VMA}{100}\right) + 3\cdot|G^*|_{binder} \left(\frac{VFA-VMA}{10,000}\right)\right]$$

$$+ (1-Pc) \left[\frac{1 - \frac{VMA}{100}}{4,200,000} + \frac{VMA}{3\cdot VFA \cdot |G^*|_{binder}}\right]^{-1},$$

(3)

where the contact area $Pc$ is defined as follows:

$$Pc = \frac{\left(20 + \frac{VFA \cdot |G^*|_{binder}}{VMA}\right)^{0.58}}{650 + \left(\frac{VFA \cdot |G^*|_{binder}}{VMA}\right)^{0.58}},$$

(4)
and

\[ \phi = -21 \left( \log P_c \right)^2 - 55 \log P_c, \tag{5} \]

where VFA is voids filled with asphalt, VMA is percent voids in the mineral aggregate and \( G_b^* \) is the complex shear modulus of the binder.

It is generally accepted that asphalt mixtures are best modelled with a viscoelastic solid model represented by a sigmoidal shape. The only model format that describes this shape to some extent is that of the Hirsch model whereas the other equations are clearly limited.

The concept of predicting phase angle from the slope of the complex modulus versus frequency was first suggested by Booij and Thoone (1982). Christensen and Anderson (1992) used the same concept in the development of the Christensen Anderson binder model phase angle calculation. Rowe (2009) presented equations using a similar basis for sigmoidal forms that can be applied to asphalt mixture analysis, as follows (Equations (6–9)). The equations can be applied to either the analysis of master curves that contain \( |E^*| \) and phase angle data or just \( |E^*| \) data alone. The works conducted by Booij and Thoone (1982), Christensen and Anderson (1992) and more recently by Rowe (2009) demonstrate that the phase angle response can be determined from a mathematical understanding of the dependency on the stiffness (either \( |G^*| \) or \( |E^*| \)) versus the frequency (\( \omega \)) using regression parameters for a sigmoidal model. Each of the regression coefficients (\( \delta, \alpha, \beta, \lambda \)) is related to the shape of the sigmoid fit to the master curve as detailed by Mensching et al. (2017). This results in a method to determine phase angle from just dynamic modulus vs. frequency data, when it is available.

Standard logistic

\[ \log |E^*| = \delta + \frac{\alpha}{1 + e^{(\beta + \gamma (\log w))}}, \tag{6} \]

Standard logistic

\[ \phi = 90 \times \frac{d\log E^*}{d\log w} = -90 \times \alpha \gamma \frac{e^{(\beta + \gamma (\log w))}}{[1 + e^{(\beta + \gamma (\log w))}]^2}, \tag{7} \]

Generalised logistic

\[ \log |E^*| = \delta + \frac{\alpha}{1 + \lambda e^{(\beta + \gamma (\log w))}^{1/\lambda}}, \tag{8} \]

Generalised logistic

\[ \phi = 90 \times \frac{d\log E^*}{d\log w} = -90 \times \alpha \gamma \frac{e^{(\beta + \gamma (\log w))}}{[1 + \lambda e^{(\beta + \gamma (\log w))}]^{1/\lambda}]. \tag{9} \]

This paper evaluates a fundamental relationship approach to determine phase angle via the slope of the log–log of the stiffness curve (from now on referred to as the slope method). The study by Rowe (2009) investigated this relationship and concluded the validity of the relationship to a large set of modified and unmodified binders, asphalt mixes and some other polymers. This paper extends the previous research to a larger set of asphalt mixtures and evaluates the reliability of the relationship to asphalt mixtures. Phase angles predicted using the slope method are compared to laboratory measurements to assess the validity of the relationship for various mixtures. Further comparisons are made with the Hirsh model, the slope method and laboratory-measured phase angle values. Finally, the implication for rheological parameters and pavement fatigue performance predictions due to the use of predicted phase angle values as opposed to lab-measured values is assessed.
Research approach and materials

Materials and testing

Three sets of independent data were used for this study; two from specimens fabricated and tested by the research team as part of ongoing projects and the third from the LTPP database. The first set of specimens includes two 9.5 mm nominal maximum aggregate size (NMAS) (C-9.5 mm and L-9.5 mm) and one 12.5 mm NMAS (C-12.5 mm) laboratory-produced mixtures. The aggregates and binders were obtained from Rhode Island Department of Transportation (RIDOT) and specimens were fabricated to replicate a range of acceptable as-built field conditions in terms of asphalt binder and air void content. The low, optimum and high levels of asphalt and air void content combinations for the three mixtures resulted in 27 specimen conditions overall. The second set test specimens were plant-produced materials obtained from New Hampshire Department of Transportation (NH DOT) projects. These included 12.5 and 19 mm NMAS mixtures containing various amounts of RAP and RAS and a virgin mixture. The last set includes three mixtures from Vermont, New Jersey and North Carolina LTPP sections. Overall, the wide selection of mixtures used in this study covers a range in as-built conditions, NMAS, laboratory versus plant production, virgin and modified binder (V), and % RAP and RAS; this provides a platform for comparing the potential effects of these parameters on phase angle predictions. Mixture information is summarised in Table 1.

Specimens fabricated and tested by the research team were compacted using a Superpave gyratory compactor and then cut and cored to test specimen dimensions of 100 × 150 mm and 100 × 130 mm for dynamic modulus and fatigue testing, respectively. Complex modulus ($E^*$) testing was performed following the test procedure provided in AASHTO T342 in load-controlled uniaxial compression mode. Testing was done at three temperatures (4.4°C, 21.1°C, and 42.2°C).

<table>
<thead>
<tr>
<th>Rhode Island (RI) mixtures</th>
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<tbody>
<tr>
<td>Mixture label</td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>L-9.5mm</td>
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<tr>
<td>C-9.5mm</td>
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<td>C-12.5mm</td>
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<th>New Hampshire (NH) mixtures</th>
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<td>Mixture Label</td>
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<tr>
<td>NH-12.5 mm, 20%RAP</td>
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<tr>
<td>NH-12.5 mm, 30%RAP</td>
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<tr>
<td>NH-12.5 mm, RAP/RAS</td>
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<tr>
<td>NH-19 mm, 20%RAP</td>
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<td>NH-19 mm, 30%RAP</td>
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<tr>
<td>NH-19 mm, RAP/RAS</td>
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<td>Virgin mixture</td>
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<th>Mixtures from LTPP</th>
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<td>NC mixture</td>
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<td>NJ mixture</td>
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<td>VT mixture</td>
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and 37.8°C) and six loading frequencies (25, 10, 5, 1, 0.5 and 0.1 Hz) on three replicate specimens using an Asphalt Mixture Performance Tester (AMPT). Axial deformation was measured using four linear variable displacement transducers (LVDTs) with a 70 mm gauge length. Strain amplitudes during tests were limited to 50–75 micro-strain to ensure the test response remains in the linear viscoelastic range. The resulting stress and strain of the final six cycle of each loading series were used to calculate the dynamic modulus and phase angle of the mixtures. Dynamic modulus is determined by dividing peak-to-peak specimen stress to strain (Equation (10)), and phase angle is obtained from the time lag between peak to peak strain and stress (Equation 11):

\[ |E*| = \frac{\sigma_{\text{amp}}}{\varepsilon_{\text{amp}}}, \]  

\[ \phi = 2\pi f \Delta t, \]

where \( \sigma_{\text{amp}} \) is the amplitude of applied sinusoidal stress, \( \varepsilon_{\text{amp}} \) is the amplitude of sinusoidal strain response, \( f \) is the stress and strain frequency and \( \Delta t \) is the average time lag between stress peak and strain peak at a given frequency and temperature.

Cyclic fatigue testing using AMPT was carried out on specimens following the test procedure in AASHTO TP 107. Four specimens were tested at four different peak-to-peak on-specimen strains to cover the appropriate range of number of cycles to failure.

**Dynamic modulus and phase angle master curve construction from measured data**

In this study, the RHEA ™ software (Rowe & Sharrock, 2000) is used to construct \( |E*| \) and \( \phi \) master curves from measured \( |E*| \) and \( \phi \) data points using the time–temperature superposition principle. For tests performed in the linear viscoelastic range, the time–temperature superposition principle allows test isotherms to be shifted to a required temperature at a reduced frequency. The shifting in the RHEA ™ software is done following the work done by Gordon and Shaw (1994). The storage modulus, representing the elastic behaviour, and loss modulus, representing the viscous behaviour, are shifted separately. Then the shifting from the two components is averaged to produce the final shift factor. In the absence of phase angle data, the shifting in the programme is done only once using the dynamic modulus component. The same shift factors are applied to the corresponding phase angle measured points to produce phase angle master curves. The five-parameter, generalised sigmoidal model (Richards curve) was used to fit the dynamic modulus and phase angle master curves (Equations 8 and 9).

![Figure 1. Measured \( |E*| \) raw data.](image-url)
Figure 2. (a) $|E^*|$ master curves, (b) time–temperature shift factors, (c) phase angle master curves.

The $|E^*|$ and $\phi$ master curve construction process is illustrated in Figures 1 and 2. Figure 1 shows lab-measured $|E^*|$ isotherms at test temperatures of 4.4°C, 21.1°C and 37.8°C and loading frequencies of 25, 10, 5, 1, 0.5 and 0.1 Hz. The isotherms are shifted horizontally to a reduced frequency to construct the $|E^*|$ master curve at a reference temperature of 21.1°C (Figure 2(a)). The shifted points are fitted with a generalised logistic equation (Figure 2(a)). The time–temperature shift factors are shown in Figure 2(b). The same shift factors (Figure 2(b)) are applied to measured phase angle points to construct phase angle master curves (Figure 2(c)). The measured phase angle master curve is fitted with a generalised logistic equation (Figure 2(c)).

To determine mixture phase angle using the slope method, first the $|E^*|$ master curve is constructed using average measured $|E^*|$ data from replicates and $|E^*|$ values deviating from the...
mean by one standard deviation. This is done to account for some degree of variability that exists during $|E^*|$ measurement. The sample standard deviation for each replicate at each test temperature and frequency was calculated to obtain the high and low range of the measured $|E^*|$. For each set of measured data (average, average $+1$ standard deviation and average $-1$ standard deviation), independent time–temperature shifting was conducted and this yielded three master curves for $|E^*|$. These are labelled as “Measured”, “Measured High Range” and “Measured Low Range” (Figure 2(a)).

**Phase angle prediction from the slope of the log–log $|E^*|$ master curve**

Using the fitting sigmoidal logistic function, 28 $|E^*|$ master curve points were computed at 5 equally spaced points per decade on a logarithmic scale over a frequency range of 0.001–10,000 Hz. Figure 3(a) shows the points determined in this manner for average $|E^*|$, low and high range $|E^*|$. The unfitted master curve points which correspond to the unfitted phase angle points in Figure 2(b) are also shown in Figure 3(a). The phase angle points (Figure 3(b)) are determined from the slope of the log–log $|E^*|$ master curve of Figure 2(a) using Equation (9). Figure 3(b) shows the predicted phase angle from average, low and high ranges. The measured unfitted and fitted curves are shown to allow visual comparison between measured and predicted phase angle points (Figure 3(b)).

**Phase angle Prediction using the Hirsch Model**

In this portion of the study, the Hirsch model is used to determine the phase angle from binder shear modulus $|G^*|$ and mix volumetrics (Equation (5)) to allow comparison to the slope method.
prediction and lab measurements. Lab-measured $|G^*|$ binder values were used for the NH virgin mixture. However, the $|G^*|$ values back calculated using Equations (3) and (4) are used for the LTPP mixtures because the binder shear modulus values were not available. The back-calculated shear modulus is then used in Equation (5) to determine the phase angle. It should be noted that for the phase angle prediction done in this manner, the compounding error from each model could contribute to the differences or similarities observed with the Hirsch method, slope method and lab measurement comparison for the LTPP mixtures.

**Statistical analysis**

Statistical analysis is performed to examine the accuracy of the phase angle estimation from the slope of the log–log $|E^*|$ master curve. First, the Pearson linear correlation coefficient is used to measure the strength of linear association between the 28 measured and predicted data points corresponding to each mixture condition. Next, a linear relationship is fitted to the points and the equation of the fitted line is generated to allow comparison on a unity plot ($x = y$). Further statistical analysis is employed by calculating the root mean square error (RMSE) between measured and predicted phase angle points. RMSE complements the correlation coefficient because it provides the difference in measurements between each measured and predicted points. The RMSE for individual mixtures is normalised by dividing the total RMSE by the number of points.

**Impact on pavement performance evaluation**

**Mixture Black Space diagram**

Mixture Black Space diagrams assess the stiffness and relaxation capability of mixtures from a plot of $|E^*|$ versus phase angle. A recent study by Mensching et al. (2017) evaluated the correlation between certain Black Space points and low temperature cracking performance based on the Glover–Rowe (G–R) binder cracking parameter. The G–R parameter correlates binder Black Space points to non-load-associated cracking (King et al., 2012; Rowe et al., 2013). The temperature and frequency combinations evaluated for mixture Black Space points were 15°C and 0.005, 5 and 500 rad/s. For this study, the Black Space points are generated using measured and predicted phase angle data to evaluate the relative location of the points.

**Pavement fatigue life evaluation, SVECD and LVECD**

In recent studies the SVECD approach (Underwood, Baek, & Kim, 2012) has been used to evaluate the fatigue performance of various mixtures using uniaxial cyclic fatigue testing. In this approach, two relationships are developed to characterise the fatigue performance of the mixtures and are used in subsequent modelling and pavement performance prediction. The damage characteristic curve ($C$ versus $S$) is represented by the reduction in the pseudo secant modulus (material integrity) and accumulated damage during cyclic loading. The curve is fitted with an exponential function (Equation (12)). This relationship shows how the stiffness, or integrity, of the material changes as micro cracks grow during continued load applications.

The fundamental energy-based failure criterion, $G^R$ (the rate of change of the average released pseudo strain energy), is plotted versus the number of cycles to failure and demonstrates that if damage is accumulating faster, material failure will occur sooner. The curve from this relationship is fitted with Equation (13). The fitting parameters $a$, $b$, $\gamma$ and $s$ in Equations (12 and 13) are referred to as damage model coefficients. Both relationships require dynamic modulus and phase angle master curves for the linear viscoelastic characterisation of the asphalt mixtures:

$$C = e^{-as^b},$$  \hspace{1cm} (12)
\[ N_f = (G^R)^s, \]  

where \( a, b, \gamma \) and \( s \) are fitting parameters.

The LVECD programme employs a finite element structural analysis for pavement response computation and pavement fatigue performance prediction based on the SVECD approach (Eslaminia, Thirunavukkarasu, Guddati, & Kim, 2012). The fatigue coefficients from SVECD analysis are used in the model to determine the fatigue performance of the study mixtures in a pavement structure in terms of number of failure points. Failure of an element is defined when the ratio of applied loading cycles to the failure loading cycle is equal to 1 \( (N/N_f = 1) \), in which case the asphalt element is considered completely cracked. A spatial distribution plot from LVECD analysis displays the ratio of \( N/N_f \) for the matrix of 11 by 101 finite element nodes producing a total of 1111 nodes along the pavement cross-section. For this study, the number of failure points was tallied to compare relative fatigue performance.

For this study, SVECD analysis and LVECD pavement simulations were performed using both measured and predicted phase angles. The SVECD approach was used to analyse the fatigue test results and determine the damage characteristics curves \( (C \text{ versus } S \text{ and } G^R \text{ versus } N_f) \) for the study mixtures. One of the required inputs to the SVECD analysis is phase angle measurement of mixtures at three temperatures and six frequencies for linear viscoelastic characterisation of mixtures. The phase angle prediction from the stiffness data was used to obtain these values. The fatigue coefficients obtained from SVECD analysis using predicted and measured phase angle values are used in LVECD to compute the fatigue performance of the mixtures in a pavement structure. A typical pavement structure and traffic were used in the analysis. Fatigue cracking performance evaluation in terms of \( C \text{ versus } S \text{ and } G^R \text{ versus } N_f \) as well as number of failure points determined using the predicted and measured phase angle values for the mixtures were compared. This is done to assess the potential effect of predicted phase angle on the damage characteristic curves and fatigue performance prediction and further gauge the accuracy of the prediction approach in the context of pavement performance estimates.

**Results and discussion**

The results from statistical analysis, rheological indices and pavement performance evaluation are discussed in this section. Results for all of the mixtures evaluated are summarised in tabular form and example figures for two cases were chosen for illustration. The two cases represent the best and worst examples, from the perspective of match between measured and predicted phase angles.

Figure 4 demonstrates the \( |E^*| \) master curve fitted with the generalised logistic equation for Measured (Mean), measured high range (Mean + 1 Standard deviation) and measured low range (Mean – 1 Standard deviation). Comparing all Rhode Island and New Hampshire Mixtures, the L-9.5 mm 6.8AC 7AV mixture exhibited the lowest variability (Figure 4(a)) whereas the C-12.5 mm 6.1AC 7AV showed the highest variability (Figure 4(b)) in \( |E^*| \) measurement among replicates. Considering highest variability observed, it can be noted that deviation in dynamic modulus measurements among replicates is slight (CV = 11%). Therefore, the effect on phase angle prediction due to \( |E^*| \) measurement variation is also expected to be minimal.

**Comparison of measured and predicted phase angles**

Figure 5 shows the different measured and predicted phase angle master curves. The actual measured phase angle values are shown along with the logistic curve fitted to the measured points.
to illustrate the variability in phase angle measurements and its influence on the shape of the fitted logistic curve. Throughout the study, the statistical and other comparisons are done using fitted measured phase angle values as opposed to predicted values. So, it should be noted that the fitting might magnify or reduce the differences between measured and predicted values.

The phase angle master curve predicted from the measured $|E^*|$ curve is shown in Figure 5. The mixtures presented represent the best and worst matches between measured and predicted values. The L-9.5 mm 5.9AC 7AV and NH-12.5 mm, 30% RAP mixtures had the lowest differences among the Rhode Island and New Hampshire mixtures, respectively (Figure 5(a,b)). The C-12.5 mm 6.1AC 7AV and NH-19 mm, RAP/RAS mixtures exhibited the largest differences between measured and predicted values (Figure 5(c,d)). The difference between measured and predicted values appears to be higher at the peak phase angle, which corresponds to higher testing temperature or lower loading frequency. As it can be seen from actual phase angle measurements, the variability in lab measurements is usually higher at this location as well. This raises the question about the accuracy of phase angle measurements and how to incorporate the error that is present during model verification. It has to be noted that in both best and worst scenarios, the measured and predicted phase angle master curves have comparable shape and peaks at similar frequencies. This is further quantified by computing the inflection point frequency for both curves, shown in Figure 6. Overall, the inflection points match very well which is indicated by the high $R^2$ value (0.98) and low average RMSE (0.02 Hz).

The statistical analysis for Rhode Island and New Hampshire mixtures is presented in Table 2, and the relationship is shown graphically in Figure 6. A strong association between measured and predicted values is observed for the L-9.5 mm and C-9.5 mm mixture whereas the C-12.5 mm and
New Hampshire mixtures appear to have the least agreement between measured and predicted values. Overall, the prediction appears to generate lower phase angle values as compared to measured values. This is indicated by the consistent deviation of the points to the lower side of the equality line (Figure 7).
Generally, the predicted and measured phase angles are in good agreement in terms of shape of the master curve and magnitude for Rhode Island L-9.5 mm and C-9.5 mm mixtures with 7% and 8% deviation from the equality line and 1.34 and 1.62 RMSE, respectively. For the 18 mixture conditions under the 2 sets of mixtures, the average difference between measured and predicted values is less than 2°. However, a larger difference is observed in New Hampshire mixes followed by the Rhode Island C-12.5 mm with RMSE 4.24 and 4.96 and 16% and 17% deviation from the equality line, respectively. From the observation, it appears that the differences between measured and predicted values are consistently lower for certain sets of mixtures and are higher for other sets of mixtures despite the same lab measurement procedure and prediction method employed. Particularly, the composition in terms of aggregate and binder used for the C-12.5 mm mix was similar to the C-9.5 mm mix and both mixtures used identical production procedure and equipment, environmental chamber and AMPT. Hence, the higher difference observed in the C-12.5 mm could not be explained with respect to any of these parameters. However, for the C-9.5 mm and L-9.5 mm mixtures, for which measured and predicted phase angles are in better agreement (both with low average RMSE of 1.34 and 1.62), spring-loaded linear variable displacement transducers (LVDTs) were used to measure specimen deformation. However, for the C-12.5 mm and New Hampshire mixtures where the differences are higher, loose core LVDTs were used.

For all study mixtures, the same AMPT device was used. There were no differences in calibration, machine compliance, algorithm and PID parameters used. The only difference as stated above was the type of LVDT that was used for deformation measurements. A comparative study done by Lacroix (2013) showed that the measurements from spring-loaded LVDTs are less variable (within the tolerance given by the manufacturer) as compared to loose core LVDTs in a situation where both are calibrated. The study also suggested that the alignment between LVDT and the specimen causes a higher discrepancy in measurement of displacement and phase angle for loose core models as compared to spring-loaded ones. Based on the results presented here as well as from observations by others, the type of LVDTs used could be one of the reasons for higher differences between measured and predicted phase angle values observed.

The prediction of phase angle from dynamic modulus relies upon the assumption of linear viscoelastic behaviour in the small strain region and that materials can be considered thermorheologically simple and time–temperature supposition is valid. However, it has been known for many years that a true representation of asphalt mixtures over the entire range of temperature...
Table 2. Statistical evaluation.

<table>
<thead>
<tr>
<th>RI mixtures</th>
<th>AC–AV</th>
<th>$R^2$</th>
<th>Trend line equation</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>L-9.5mm</td>
<td>5.9–4</td>
<td>1.00</td>
<td>$y = 0.93x$</td>
<td>1.43</td>
</tr>
<tr>
<td></td>
<td>5.9–7</td>
<td>1.00</td>
<td>$y = 0.96x$</td>
<td>0.85</td>
</tr>
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<td></td>
<td>5.9–9</td>
<td>1.00</td>
<td>$y = 0.94x$</td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>6.3–4</td>
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<td>$y = 0.93x$</td>
<td>1.30</td>
</tr>
<tr>
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<td>6.3–7</td>
<td>0.99</td>
<td>$y = 0.94x$</td>
<td>1.69</td>
</tr>
<tr>
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<td>6.3–9</td>
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<td>$y = 0.91x$</td>
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</tr>
<tr>
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<td>6.8–4</td>
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<td>$y = 0.95x$</td>
<td>1.09</td>
</tr>
<tr>
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<td>1.00</td>
<td>$y = 0.94x$</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>6.8–9</td>
<td>1.00</td>
<td>$y = 0.95x$</td>
<td>1.14</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>94% (6%)</td>
<td>1.34</td>
</tr>
<tr>
<td>C-9.5mm</td>
<td>5.9–4</td>
<td>0.99</td>
<td>$y = 0.94x$</td>
<td>1.26</td>
</tr>
<tr>
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<td>0.96</td>
<td>$y = 0.93x$</td>
<td>1.31</td>
</tr>
<tr>
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<td>$y = 0.92x$</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>6.3–4</td>
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<td>$y = 0.93x$</td>
<td>1.51</td>
</tr>
<tr>
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<td>1.46</td>
</tr>
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<td>$y = 0.92x$</td>
<td>1.62</td>
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<tr>
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<td>1.73</td>
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<td>1.95</td>
</tr>
<tr>
<td></td>
<td>6.8–9</td>
<td>0.86</td>
<td>$y = 0.91x$</td>
<td>2.15</td>
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<tr>
<td>Average</td>
<td></td>
<td></td>
<td>92% (8%)</td>
<td>1.62</td>
</tr>
<tr>
<td>C-12.5mm</td>
<td>5.2–3</td>
<td>0.98</td>
<td>$y = 0.86x$</td>
<td>3.57</td>
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<tr>
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<td>5.2–7</td>
<td>0.96</td>
<td>$y = 0.82x$</td>
<td>4.84</td>
</tr>
<tr>
<td></td>
<td>5.2–9</td>
<td>0.89</td>
<td>$y = 0.87x$</td>
<td>2.91</td>
</tr>
<tr>
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<td>5.6–5</td>
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<td>$y = 0.87x$</td>
<td>3.41</td>
</tr>
<tr>
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<td>5.6–7</td>
<td>0.93</td>
<td>$y = 0.82x$</td>
<td>5.98</td>
</tr>
<tr>
<td></td>
<td>5.6–9</td>
<td>0.97</td>
<td>$y = 0.81x$</td>
<td>4.99</td>
</tr>
<tr>
<td></td>
<td>6.1–4</td>
<td>0.83</td>
<td>$y = 0.82x$</td>
<td>5.59</td>
</tr>
<tr>
<td></td>
<td>6.1–7</td>
<td>0.77</td>
<td>$y = 0.73x$</td>
<td>8.65</td>
</tr>
<tr>
<td></td>
<td>6.1–9</td>
<td>0.95</td>
<td>$y = 0.84x$</td>
<td>4.73</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>83% (17%)</td>
<td>4.96</td>
</tr>
<tr>
<td>NH mixtures</td>
<td>NMAS</td>
<td>$R^2$</td>
<td>Trend line equation</td>
<td>RMSE</td>
</tr>
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<td>0.86</td>
<td>$y = 0.88x$</td>
<td>3.05</td>
</tr>
<tr>
<td>NH 12.5 mm, 30%RAP</td>
<td>12.5</td>
<td>0.99</td>
<td>$y = 0.89x$</td>
<td>2.40</td>
</tr>
<tr>
<td>NH 12.5 mm, RAP/RAS</td>
<td>12.5</td>
<td>0.99</td>
<td>$y = 0.82x$</td>
<td>4.36</td>
</tr>
<tr>
<td>NH 19 mm, 20%RAP</td>
<td>19</td>
<td>0.99</td>
<td>$y = 0.83x$</td>
<td>4.45</td>
</tr>
<tr>
<td>NH 19 mm, 30%RAP</td>
<td>19</td>
<td>0.94</td>
<td>$y = 0.83x$</td>
<td>5.14</td>
</tr>
<tr>
<td>NH 19 mm, RAP/RAS</td>
<td>19</td>
<td>0.97</td>
<td>$y = 0.78x$</td>
<td>6.03</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>84% (16%)</td>
<td>4.24</td>
</tr>
</tbody>
</table>

and frequency requires the use of viscoelastic-plastic models. The plastic strain that occurs is largely a function of movement within the aggregate skeleton and represents the complex nature of a mixture rather than a bituminous binder. Work conducted on mixture visco-plastic behaviour suggests that the plastic deformation occurs at very low stress and most likely can be considered as a stress-dependent zero yield plasticity (Darabi, Al-Rub, Masad, Huang, & Little, 2011; Drescher, Kim, & Newcomb, 1993; Rowe & Pellinen, 2004). If the response to load is considered a simple model – zero yield stress-dependent linear plastic strain – the strain produced by a sinusoidal stress application will be completely out of phase with the load response in a similar manner to a viscous strain. The stress imposed upon the aggregate skeleton will be higher when the binder is less stiff, associated with lower frequencies or higher temperatures. Thus,
the measured phase angle of a mixture (which includes a plastic response) is always theoretically going to be greater than that of a mixture represented only by viscoelastic behaviour. Thus, when a viscoelastic consideration is applied to the calculation of phase angle from the complex modulus of a mixture a bias in the data is expected.

It should be noted that the calibration of the phase angle measurement is complex. Phase angle is not a direct measurement but rather a calculation from collected data. The system in use was verified to deliver a zero-phase angle with an aluminium specimen fixed in the device. However, at higher frequencies some bias in measurement may exist for reasons not immediately intuitive to the researchers. While magnitudes of these effects are not known, they may exist and are mentioned in this paper for completeness of this discussion. If these effects are insignificant it should be possible to assess the plastic strain effects compared to the various models that exist.

**Comparison of the slope-based prediction method with the Hirsh model prediction**

A virgin mixture is used to compare phase angle prediction among the slope-based method, Hirsch model and lab measurement. A virgin mixture is used for this comparison because the binder modulus is a direct representation of the material in the mixture; this is not possible with
RAP mixtures due to unknown degree of blending between the RAP and virgin binders in the mixture. Phase angle master curves from the slope method, Hirsch model and lab measurement and the relationship between them are shown graphically in Figure 8. The lab-measured and predicted phase angles with the slope method are in better agreement in terms of shape and magnitude than those from the Hirsch model which is indicated by the lower RMSE (Figure 8(c)). The Black Space diagram (Figure 8(b)) also supports the above observation. While the Hirsch model has been found useful, it tends to follow a certain defined shape (as seen in the figure) and does not show the same level of flexibility that is observed with the slope-based method.

The mixture data from the LTPP sections are used to produce phase angle master curves using the slope method and the Hirsch model. Although the predictions cannot be verified...
against lab measurements, the presented plots show some of the possible differences that could be encountered due to the use of one method over the other. For the North Carolina mixture, (Figure 9(a)) the master curve peaks do not match and larger differences are observed in terms of magnitude. The Vermont mixture master curves from both methods are relatively similar in terms of shape and peak location but a larger difference is observed in the magnitude of the peak point (Figure 9(b)). The New Jersey mixture master curves follow distinctly different shapes and the peak points do not match (Figure 9(c)). It should be noted here that the Hirsch model predictions here utilised binder $|G*|$ values that were predicted from $|E*|$ values. Once again due to lack of measured phase angle data, it is difficult to comment as to which method

Figure 9. Phase angle predicted using the slope method and the Hirsch model: (a) NC, (b) VT, (c) NJ.
provides more accurate predictions and the observations made here are purely comparative in nature.

**Impact on pavement performance evaluation**

**Mixture Black Space diagram**

The mixture Black Space diagrams produced using measured and predicted phase angles are shown in Figure 10. These plots correspond to the best and worst matches between lab measurement and slope prediction among C-9.5 mm and L-9.5 mm mixtures. Since the $|E^*|$ values are the same, the difference due to the use of measured versus predicted phase angles is a shift to the right or the left. For the best fit the measured Black Space points are shifted to the left very slightly. For the poorest fit, there is a larger difference between the points, with the largest difference at 500 rad/s.

**Pavement fatigue life evaluation, SVECD and LVECD**

The damage characteristic curves ($C$ versus $S$) and the fatigue failure criterion curves ($G^R$ versus $N_f$) are generated using measured and predicted phase angle values and are shown in Figures 11 and 12, respectively. The L-9.5 mm 5.9AC 7AV and C-12.5 mm 6.1AC 7AV mixtures presented here represent the best and worst matches. The damage characteristic and failure criterion curves are almost identical for the best match case. The worst match case shows slightly different damage characteristic curves and a shifted failure criterion curve. The values of the model parameters ($a$, $b$, $r$ and $s$) show how similar the best curves are and the difference in the worst fit.

Figure 13 shows the predicted number of failure points in the pavement over 20 years of service for the L-9.5 mm 5.9AC 7AV (best match among phase angle values) and C-12.5 mm 6.1AC
7AV (worst match among phase angle values) mixtures from the LVECD pavement evaluation. As expected, the pavement simulations for the mixture with the least difference between measured and predicted values generated nearly identical curves. The mixture with the worst match between measured and predicted phase angles shows a difference of approximately 25 failure points at the end of the analysis period (20 years); this translates to a decrease of approximately 25% in the number of failure points with the predicted values. However, when this is translated to the percentage of failed points in the overall structure, the difference is only 2% (7% using the predicted values and 9% using the measured values). Since the LVECD model is not calibrated with field performance data, it is difficult to translate the differences to actual field cracking and evaluate whether this magnitude of difference would change any decisions that would be made with respect to using this mixture or design.

Generally, mixtures where the spring-loaded LVDT is used exhibited less variation among measured and predicted values and, hence, with missing phase angle data, the phase angle obtained from the slope-based method can be used for linear viscoelastic characterisation of mixes in the SVECD and LVECD models to obtain a comparable fatigue performance prediction.

**Summary and conclusion**

In this study, a fundamental relationship to determine the phase angle from the slope of the log–log of the $|E^*|$ master curve is evaluated using three sets of independent mixtures. The
validity of the method is investigated by comparing predicted values with lab measurements. Further comparison is performed between lab measurements, the slope method and Hirsch model values. Statistical quantities are calculated to examine the accuracy of the phase angle estimations. Rheological parameters are generated using measured and predicted values and compared. Finally, fatigue characterisation using the SVECD approach and pavement evaluation using the LVECD model is conducted using measured and predicted phase angle values to assess the implication of using one over the other in fatigue performance prediction. The following conclusions are drawn based on the observations:

- Overall, the variability in $|E^*|$ measurement between replicates was low. Due to this the predicted phase angle values from average $|E^*|$, low and high ranges $|E^*|$ were very close.
- For specimens where the lab measurements were done using spring-loaded LVDTs, measured phase angle values match very well with values predicted using the slope method with an average difference of less than 2°. However, a larger difference (up to an average difference of $5^\circ$) is observed for specimens that used loose core LVDTs for measurement. The highest differences are observed around the peak values where the phase measurements are also more variable due to high test temperatures or low test frequencies. Further study is needed to determine the exact attribution of measurement inaccuracy on the observed differences due to the type of LVDT and other factors.
Figure 13. LVECD pavement simulation with predicted and measured phase angles.

- Generally, the predicted phase angles from the slope method are consistently lower than lab-measured values. Two hypotheses were presented as to why this might be observed for the majority of mixtures. The first is that neglecting the plastic response that might be present at lower frequencies and higher temperatures may cause the measured phase angle to be higher than it actually is. The second reason could be attributed to the complexity in phase angle calibration. The calibration performed at zero-phase angle might cause bias during lab measurement of phase angle at different temperatures and frequencies.

- The phase angle master curves constructed from lab measurements and predicted using the slope method follow a comparable shape and exhibit very similar inflection points.

- Though extensive study has not been done for comparing lab measurement, the Hirsch model and the slope method, it is observed that measured and slope method values agree very well in terms of master curve shape and magnitude as compared to the Hirsch model. The same interpretation applies for the Black Space diagram generated from the three methods. The comparison between the Hirsch and slope methods using LTPP data showed the possible differences that arise due to the use of one method over the other. At different instances the phase angle curves were different in magnitude, shape and peak point location. Generally, it is observed that the Hirsch model lacks flexibility in terms of shape due to the underlying functional form of the model.

- The mixture Black Space points generated using both measured and predicted phase angles from the slope method are comparable.

- For the two set of mixtures corresponding to best match among predicted and measured phase angle values, the damage characteristics curves from SVECD and number of failure points from LVECD were similar, indicating the phase angle predicted from the slope
method can be used when no phase angle data are present for the linear viscoelastic characterisation of mixtures without affecting the results.

The advancement in pavement performance mechanistic models calls for better accuracy in material characterisation. The estimation of phase angle from measured stiffness using the slope method is viable due to the availability and growing reliability of stiffness measurements with advancements in equipment as compared to phase angle measurements. With the availability of such a simple and robust method, the phase angle can be computed from $|E^*|$ master curves for mixtures for which phase angle measurements are missing. This also largely applies to LTPP data. Subsequently, rheological indices which require phase angle can be generated from historic data and can be calibrated with available field performance data. The study also gives insight into some bias that exists during lab measurements.

Since the validation of the prediction of phase angle from the slope of the log–log curve is done by comparing with lab-measured values, any bias on lab measurement presents the same bias on the validation. Future work is needed to identify the magnitude of bias. Moreover, the binder grades evaluated in this study are limited to PG 58-28 and PG 64-28 binders that are extensively used in the Northeastern part of the United States. Future study is recommended to evaluate the influence of binder grade on the prediction capability of the proposed method. Furthermore, future study should look at the potential effect of permanent strain that could be encountered during complex modulus testing (acceptable up to 1500 $\mu$ according to AASHTO T342). Finally, the validity of the method to polymer-modified and aged mixtures should be investigated by employing the method described in this study to determine if the modification and aging alter the relationship between dynamic modulus and phase angle.

Acknowledgments
The authors would like to acknowledge New Hampshire and Rhode Island Departments of Transportation for providing the materials for this study. Acknowledgement is also extended to Katie Haslett for her assistance with specimen preparation at the University of New Hampshire.

Disclosure statement
No potential conflict of interest was reported by the authors.

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